AP® CALCULUS AB
2009 SCORING GUIDELINES (Form B)

Question 1

At a certain height, a tree trunk has a circular cross section. The radius $R(t)$ of that cross section grows at a rate modeled by the function

$$\frac{dR}{dt} = \frac{1}{16}\left(3 + \sin\left(t^2\right)\right) \text{ centimeters per year}$$

for $0 \le t \le 3,$ where time t is measured in years. At time $t = 0,$ the radius is 6 centimeters. The area of the cross section at time t is denoted by $A(t)$.

(a) Write an expression, involving an integral, for the radius $R(t)$ for $0 \le t \le 3.$ Use your expression to find $R(3)$.

(b) Find the rate at which the cross-sectional area $A(t)$ is increasing at time $t = 3$ years. Indicate units of measure.

(c) Evaluate $\int_0^3 A'(t)\,dt.$ Using appropriate units, interpret the meaning of that integral in terms of cross-sectional area.

(a) $R(t) = 6 + \int_0^t \frac{1}{16}\left(3 + \sin\left(x^2\right)\right) dx$ $R(3) = 6.610$ or 6.611	$3 : \begin{cases} 1 : \text{integral} \\ 1 : \text{expression for } R(t) \\ 1 : R(3) \end{cases}$
(b) $A(t) = \pi\left(R(t)\right)^2$ $A'(t) = 2\pi R(t) R'(t)$ $A'(3) = 8.858 \text{ cm}^2/\text{year}$	$3 : \begin{cases} 1 : \text{expression for } A(t) \\ 1 : \text{expression for } A'(t) \\ 1 : \text{answer with units} \end{cases}$
(c) $\int_0^3 A'(t)\,dt = A(3) - A(0) = 24.200$ or 24.201 From time $t = 0$ to $t = 3$ years, the cross-sectional area grows by 24.201 square centimeters.	$3 : \begin{cases} 1 : \text{uses Fundamental Theorem of Calculus} \\ 1 : \text{value of } \int_0^3 A'(t)\,dt \\ 1 : \text{meaning of } \int_0^3 A'(t)\,dt \end{cases}$

CALCULUS AB

SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.

Work for problem 1(a)

$R(t) = \int_0^t \frac{1}{16}(3+\sin(x^2))\,dx + 6$ for $0 \le t \le 3$

Thus $R(3) = \int_0^3 \frac{1}{16}(3+\sin(x^2))\,dx + 6 = 0.611 + 6 = 6.611$

Work for problem 1(b)

$A'(t) = \frac{dA}{dt} = \frac{dA}{dR} \cdot \frac{dR}{dt} = 2\pi R(t) \cdot \frac{1}{16}(3+\sin(t^2)) = \frac{\pi R(t)}{8}(3+\sin(t^2))$

Hence at $t=3$ years, $A'(3) = \frac{\pi R(3)}{8}(3+\sin(3^2)) = 8.858$

The unit of measure is centimeter²/year.

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Continue problem 1 on page 5.

Work for problem 1(c)

$\int_0^3 A'(t)\,dt = A(t)\Big|_0^3 = A(3) - A(0) = \pi(R(3))^2 - \pi(R(0))^2$

$= 24.207$

The unit is cm^2, and this intergral means the growth of the area of the cross section from year 0 to year 3.

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CALCULUS AB

SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.

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Work for problem 1(a)

$$\int dR = \frac{1}{16} \int (3 + \sin t^2)\,dt$$

$$R = \frac{1}{16} \int (3 + \sin t^2)\,dt + 6$$

$$R(3) = \frac{1}{16} \int_0^3 (3 + \sin t^2)\,dt + 6 = 6.611 \text{ cm}$$

Work for problem 1(b)

$$A(t) = \pi r^2 = \pi (R(t))^2$$

$$\frac{dA}{dt} = 2\pi\, R(t) \cdot \frac{dR}{dt}$$

$$\left.\frac{dA}{dt}\right|_3 = 2\pi \cdot R(3) \cdot \left.\frac{dR}{dt}\right|_3 = 8.858 \text{ cm}^2/\text{yr}$$

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Continue problem 1 on page 5.

Work for problem 1(c)

$\int_0^3 A'(t)dt = \int_0^3 \frac{dA}{dt} = A(3) - A(0)$

$A(3) = \pi(R(3))^2 = 137.298$

$A(0) = \pi(6)^2 = 36\pi$

$A(3) - A(0) = 24.201 \text{ cm}^2$

$\int_0^3 A'(t)dt = 24.201 \text{ cm}^2$ is the ~~a~~ cross-sectional area of the trunk at $t = 3$ yrs.

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CALCULUS AB

SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.

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Work for problem 1(a)

$\frac{dR}{dt} = \frac{1}{16}(3 + \sin(t^2)) \Rightarrow dR = \left(\frac{1}{16}(3 + \sin t^2)\right) dt$

$R(t) = \int_0^3 \left[\frac{1}{16}(3 + \sin t^2)\right] dt \Rightarrow R(3) - R(0) = \int_0^3 \left[\frac{1}{16}(3 + \sin t^2)\right] dt$

$R(3) - 6 = 0.611 \Rightarrow R(3) = 6.611$ centimeters

Work for problem 1(b)

$A = \pi r^2$; $r = R(t)$

$\frac{dA}{dr} = \frac{dA}{dt} \cdot \frac{dt}{dr} \Rightarrow \frac{dA}{dt} = \frac{dA}{dr} \cdot \frac{dr}{dt} \Rightarrow \frac{dA}{dt} = 2\pi r \cdot \left(\frac{1}{16}(3 + \sin t^2)\right)$

when $t = 3 \Rightarrow \frac{dA}{dt} = 2\pi(3)\left(\frac{1}{16}(3 + \sin 3^2)\right) = 4.020 \frac{\text{cm}^2}{\text{year}}$

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Continue problem 1 on page 5.

Work for problem 1(c)

$$A'(t) = \frac{dA}{dt} = \frac{\pi r}{8}(3 + \sin t^2) \Rightarrow \int_0^3 A'(t)\,dt = \int_0^3 \frac{\pi r}{8}(3+\sin t^2)\,dt$$

$= 5.677 \text{ cm}^2$

this integral shows us the difference between the cross-sectional area In the first 3 years, in other words it shows us the increase of the cross-sectional Area in the first 3 years. in cm^2.

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Question 1

Sample: 1A
Score: 9

This student earned all 9 points. Note that in part (b) the student earned the $A(t)$ point implicitly. In part (c) the student earned the answer point in spite of using a rounded value for $R(3)$ from part (a).

Sample: 1B
Score: 6

The student earned 6 points: 1 point in part (a), 3 points in part (b), and 2 points in part (c). In part (a) the student only earned the point for $R(3)$ since an integral expression for $R(t)$ is not included. In part (b) the student's work is correct. In part (c) the student earned the first 2 points. The student did not earn the point for the meaning of the definite integral since the response mentions cross-sectional area at a particular time rather than growth in cross-sectional area over the three-year period.

Sample: 1C
Score: 4

The student earned 4 points: 1 point in part (a), 2 points in (b), and 1 point in (c). In part (a) the student only earned the point for $R(3)$ since an integral expression for $R(t)$ is not included. In part (b) the last equality on the student's second line earned the first 2 points. Although $A(t)$ is not explicitly stated, the student earned the $A(t)$ point. The student did not earn the answer point since 3 is used, instead of $R(3)$, in the calculation of $\frac{dA}{dt}$ at $t = 3$. In part (c) the student earned the point for the meaning of the definite integral.

Question 2

A storm washed away sand from a beach, causing the edge of the water to get closer to a nearby road. The rate at which the distance between the road and the edge of the water was changing during the storm is modeled by $f(t) = \sqrt{t} + \cos t - 3$ meters per hour, t hours after the storm began. The edge of the water was 35 meters from the road when the storm began, and the storm lasted 5 hours. The derivative of $f(t)$ is $f'(t) = \frac{1}{2\sqrt{t}} - \sin t$.

(a) What was the distance between the road and the edge of the water at the end of the storm?

(b) Using correct units, interpret the value $f'(4) = 1.007$ in terms of the distance between the road and the edge of the water.

(c) At what time during the 5 hours of the storm was the distance between the road and the edge of the water decreasing most rapidly? Justify your answer.

(d) After the storm, a machine pumped sand back onto the beach so that the distance between the road and the edge of the water was growing at a rate of $g(p)$ meters per day, where p is the number of days since pumping began. Write an equation involving an integral expression whose solution would give the number of days that sand must be pumped to restore the original distance between the road and the edge of the water.

(a) $35 + \int_0^5 f(t)\,dt = 26.494$ or 26.495 meters	$2: \begin{cases} 1: \text{integral} \\ 1: \text{answer} \end{cases}$
(b) Four hours after the storm began, the rate of change of the distance between the road and the edge of the water is increasing at a rate of 1.007 $\text{meters}/\text{hours}^2$.	$2: \begin{cases} 1: \text{interpretation of } f'(4) \\ 1: \text{units} \end{cases}$
(c) $f'(t) = 0$ when $t = 0.66187$ and $t = 2.84038$ The minimum of f for $0 \le t \le 5$ may occur at 0, 0.66187, 2.84038, or 5. $f(0) = -2$ $f(0.66187) = -1.39760$ $f(2.84038) = -2.26963$ $f(5) = -0.48027$ The distance between the road and the edge of the water was decreasing most rapidly at time $t = 2.840$ hours after the storm began.	$3: \begin{cases} 1: \text{considers } f'(t) = 0 \\ 1: \text{answer} \\ 1: \text{justification} \end{cases}$
(d) $-\int_0^5 f(t)\,dt = \int_0^x g(p)\,dp$	$2: \begin{cases} 1: \text{integral of } g \\ 1: \text{answer} \end{cases}$

Work for problem 2(a)

$f(0) = 35$

$35 + \int_0^5 f(t)\,dt \approx 35 - 8.505 \approx 26.495$ m

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Work for problem 2(b)

$f'(4) = 1.007$

At 4 hours into the thunderstorm, the rate at which the distance between the road and the edge of the water was changing is increasing by 1.007 m/h².

Continue problem 2 on page 7.

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Work for problem 2(c)

$f'(t) = \frac{1}{2\sqrt{t}} - \sin t = 0$

$f'(0.662) = 0$ $\quad$ $+ \mid -$ at $.662$ $\quad$ possible Min

$f'(2.840) = 0$ $\quad$ $- \mid +$ at 2.840

$f(0) = -2$

$f(2.840) = -2.270$

$f(5) = -0.480$

Decreasing most rapidly at $t = 2.840$

Work for problem 2(d)

$\int_0^5 f(t)\,dt$ $\quad$ distance grown ≈ -8.505

$-8.505 + \int_0^t g(p)\,dp = 0$

$\int_0^t g(p)\,dp = 8.505$ m

Work for problem 2(a)

Let $F(t)$ be the antiderivative of $f(x)$.

$\rightarrow F(t) = \frac{2}{3}t^{\frac{3}{2}} + \sin t - 3t + C$.

Since $F(0) = 35$, $C = 35$.

$\cancel{F(5) = \frac{2}{3} \times 5^{\frac{3}{2}} + \sin(5) - 3 \times 5 + C = 35}$.

$\cancel{C = 43.505}$.

$\cancel{F(0) = C = \boxed{43.505m}}$

$F(5) = \frac{2}{3} \times 5^{\frac{3}{2}} + \sin(5) - 3 \times 5 + 35$

$= \cancel{43.505m} = \boxed{26.495m}$

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Work for problem 2(b)

$f(t)$ indicates the rate at which the distance between the road and the edge of the water was changing.

Therefore, $f'(t)$ indicates the rate at which the changing rate of the distance changes.

$f'(4) = 1.007$ means that the rate at which the ~~changing~~ changing rate of the distance between the road and the edge of the water is ~~1.007 meters per~~ 1.007 m/hr² when the storm lasted for 4 hours.

Continue problem 2 on page 7.

Work for problem 2(c)

The distance between the road and the edge of the water decreases most rapidly. $\Leftrightarrow$ $f(t)$ is minimum.

$f(t)$ minimum ~~when~~ at the endpoint of $[0,5]$ or at the point at which $f'(t)=0$. ~~and $f''(t)>0$~~.

$f(0)=-3$; $f(5)=-0.480$.

$f'(t)=\frac{1}{2\sqrt{t}}-\sin t=0. \Rightarrow t=0.662, 2.84_{cal}$

$f(0.662)=-1.372$.

$f(2.84)=-2.270$.

$\therefore$ minimum at $t=0$ (just when the storm started)

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Work for problem 2(d)

The distance that needs to be restored is $35-26.495=8.505$m.

$\rightarrow \boxed{\int_0^x g(p)\,dp = 8.505}$

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Work for problem 2(a)

$\cancel{\frac{dr}{dt}}$ $f(t) = \sqrt{t} + \cos t - 3$

$f'(t) = \frac{1}{2\sqrt{t}} - \sin t$ $\boxed{d = 35 \quad t = 0}$ $0 \le t \le 5$

(a) $d(5) = ?$ $\int_0^5 f(t)\,dt = d(5) - d(0)$

$= -8.50536$.

$\therefore d(5) = d(0) - 8.505$

$= \boxed{26.495 \text{ m } (3.\text{d.p})}$

Work for problem 2(b)

$f'(4) = 1.007$

$f'(4)$ means that during the fourth hour of the storm, the rate of change of 'the rate of change between the road and the edge of water' was 1.007. i.e.

Continue problem 2 on page 7.

Work for problem 2(c)

d = distance between road and water

d decreasing most rapidly

$= \cancel{\frac{d}{dt}}$ ~~negative~~ max^{m} value at $\frac{d}{dt}(d) = f'(t)$

$= f''(t) = 0$.

$\therefore t = 4.68775$ or $42.\cancel{42}4106$

but $0 \le t \le 5$

$\therefore t = 4.688$ (3.d.p.) hour.

$=$ 4 hours 41 min (nearest whole min) after storm starts.

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Work for problem 2(d)

(d) $g(p)$

Sand lost during storm $= \int_0^5 f(t)\,dt$.

~~Sand pump in~~ $\cancel{\int_0^p g(p)dp = \int_0^5 f(t)\,dt}$

~~let $S(p)$ = sand pumped in at time p.~~

$\therefore$ ~~Solution: $S(p)$~~

$\therefore \int_0^5 f(t)\,dt = \int_0^p g(p)\,dp$ for sand to be restored to initial condition.

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Question 2

Sample: 2A
Score: 9

The student earned all 9 points. Note that in part (d) the student's second line earned both points. The t variable that the student uses in the first integral was ignored. That t is in hours after the start of the storm, but the t variable in the student's second integral is in days.

Sample: 2B
Score: 6

The student earned 6 points: 2 points in part (a), 1 point in part (b), 1 point in part (c), and 2 points in part (d). In part (a) the student's work is correct. The student does not include a definite integral but earned the integral point for correct antidifferentiation, use of the initial condition, and evaluation at 5. In part (b) the student earned the units point. Since the response does not include the word "increasing," the interpretation point was not earned. In part (c) the student earned the first point for considering $f'(t) = 0.$ The student did not earn the answer point due to evaluation errors and was not eligible for the justification point. In part (d) the student's boxed equation earned both points.

Sample: 2C
Score: 3

The student earned 3 points: 2 points in part (a), no points in part (b), no points in part (c), and 1 point in part (d). In part (a) the student's work is correct. In part (b) the response does not include the word "increasing" or any units. In part (c) the student is seeking a maximum value rather than a minimum value. The student considers $f''(t) = 0$ instead of $f'(t) = 0.$ In part (d) the student earned the point for the integral of g in spite of using the same name for the upper limit of integration and the variable of integration. The answer point was not earned since the response lacks a negative sign in the integral equation.

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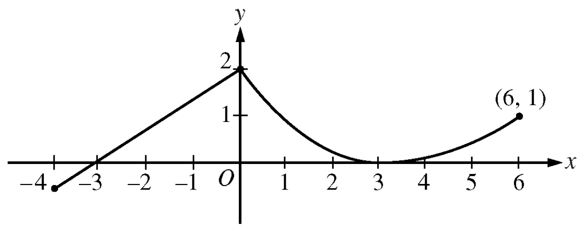
2009 SCORING GUIDELINES (Form B)

Question 3



Graph of f

A continuous function f is defined on the closed interval $-4 \le x \le 6$. The graph of f consists of a line segment and a curve that is tangent to the x-axis at $x = 3$, as shown in the figure above. On the interval $0 < x < 6$, the function f is twice differentiable, with $f''(x) > 0$.

(a) Is f differentiable at $x = 0$? Use the definition of the derivative with one-sided limits to justify your answer.

(b) For how many values of a, $-4 \le a < 6$, is the average rate of change of f on the interval $[a, 6]$ equal to 0 ? Give a reason for your answer.

(c) Is there a value of a, $-4 \le a < 6$, for which the Mean Value Theorem, applied to the interval $[a, 6]$, guarantees a value c, $a < c < 6$, at which $f'(c) = \frac{1}{3}$? Justify your answer.

(d) The function g is defined by $g(x) = \int_0^x f(t)\,dt$ for $-4 \le x \le 6$. On what intervals contained in $[-4, 6]$ is the graph of g concave up? Explain your reasoning.

(a) $\lim_{h\to 0^-} \frac{f(h) - f(0)}{h} = \frac{2}{3}$ $\lim_{h\to 0^+} \frac{f(h) - f(0)}{h} < 0$ Since the one-sided limits do not agree, f is not differentiable at $x = 0$.	$2 : \begin{cases} 1 : \text{sets up difference quotient at } x = 0 \\ 1 : \text{answer with justification} \end{cases}$
(b) $\frac{f(6) - f(a)}{6 - a} = 0$ when $f(a) = f(6)$. There are two values of a for which this is true.	$2 : \begin{cases} 1 : \text{expression for average rate of change} \\ 1 : \text{answer with reason} \end{cases}$
(c) Yes, $a = 3$. The function f is differentiable on the interval $3 < x < 6$ and continuous on $3 \le x \le 6$. Also, $\frac{f(6) - f(3)}{6 - 3} = \frac{1 - 0}{6 - 3} = \frac{1}{3}$. By the Mean Value Theorem, there is a value c, $3 < c < 6$, such that $f'(c) = \frac{1}{3}$.	$2 : \begin{cases} 1 : \text{answers "yes" and identifies } a = 3 \\ 1 : \text{justification} \end{cases}$
(d) $g'(x) = f(x)$, $g''(x) = f'(x)$ $g''(x) > 0$ when $f'(x) > 0$ This is true for $-4 < x < 0$ and $3 < x < 6$.	$3 : \begin{cases} 1 : g'(x) = f(x) \\ 1 : \text{considers } g''(x) > 0 \\ 1 : \text{answer} \end{cases}$

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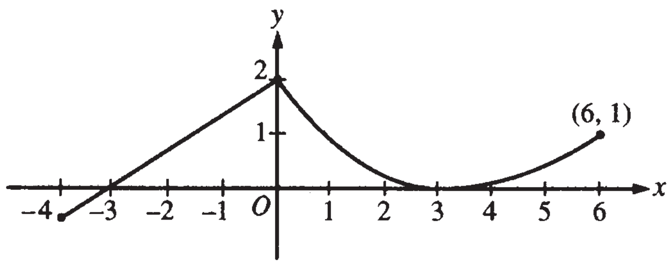

Graph of f

Work for problem 3(a)

f is differentiable at $x=0 \iff \lim_{x\to 0} \frac{f(x)-f(0)}{x-0}$ exists

Now $\lim_{x\to 0^+} \frac{f(x)-f(0)}{x} < 0$ because $f(x) < f(0)$ and $x > 0$

$\lim_{x\to 0^-} \frac{f(x)-f(0)}{x} > 0$ because $f(x) < f(0)$ and $x < 0$

so $\lim_{x\to 0^+} \frac{f(x)-f(0)}{x} \neq \lim_{x\to 0^-} \frac{f(x)-f(0)}{x}$

$\Rightarrow \lim_{x\to 0} \frac{f(x)-f(0)}{x}$ does not exist $\Rightarrow f$ is not differentiable at $x=0$

Work for problem 3(b)

Average rate of change of f on $[a, 6] = 0$

$\Rightarrow \frac{f(6)-f(a)}{6-a} = 0 \Rightarrow f(a) = f(6)$, $a \neq 6$

$\Rightarrow$ # of possible values of a = 2

Continue problem 3 on page 9.

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Work for problem 3(c)

Choose $a = 3$.

f is differentiable on ~~[3, 6]~~ $(3,6)$ and continuous on $[3,6]$.

By Mean Value Theorem, ~~(3,6) ⊆ [3,6]~~

~~∃~~ $\exists\, c \in [3,6]$ such that $f'(c) = \frac{f(6)-f(3)}{6-3}$

$= \frac{1-0}{3}$

$= \frac{1}{3}$

Thus there is a value of a that satisfies the condition, which is 3.

Work for problem 3(d)

g is concave up on (a,b)

$\Leftrightarrow g''(x) = \frac{d}{dx} g'(x) = \frac{d}{dx} f(x) = f'(x) \geq 0$ on (a,b)

$\Leftrightarrow f(x)$ is increasing on (a,b)

$f(x)$ is increasing on $(-4,0)$ and $(3,6)$

Thus g is concave up on the intervals $(-4,0)$ and $(3,6)$

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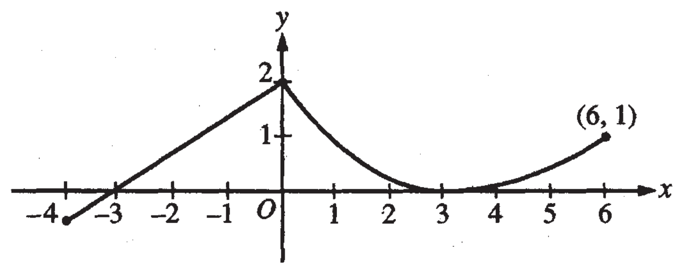
END OF PART A OF SECTION II
IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.



Graph of f

Work for problem 3(a)

No, f is not differentiable.

For $f(0)$ to be differentiable, $\lim_{x \to 0^-} f'(x) = \lim_{x \to 0^+} f'(x)$.

$f'(0^-) = \frac{2}{3}$, but $f'(0^+) = 1$.

Work for problem 3(b)

There are two values where the average rate of change of f on $[a, 6]$ equals 0. Average rate, or slope of the secant line, must equal to zero: Average rate $= \frac{1 - f(a)}{6 - a}$.

For the slope to be zero, $f(a) = 1$. There are two x values in the graph with a corresponding y value of 1.

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Continue problem 3 on page 9.

Work for problem 3(c)

Yes, there is. For the Mean value theorem, f(x) must be continuous and differentiable at [a, b]. f(x) with endpoint is continuous and differentiable at points from $x=0$ to $x=6$.
Mean Value Theorem states the following:

$$f'(c) = \frac{f(b)-f(a)}{b-a} = \frac{1}{3}$$

$$= \frac{1-f(a)}{6-a} = \frac{1}{3}$$

At $a=3$, $\frac{1-0}{6-3} = \frac{1}{3}$.

Work for problem 3(d)

For g(x) to be concave up, $g''(x) > 0$.

$g''(x) = f'(x) > 0$.

$f'(x) > 0$ on the intervals $[-4,2]$ and $[3,6]$.

END OF PART A OF SECTION II
IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

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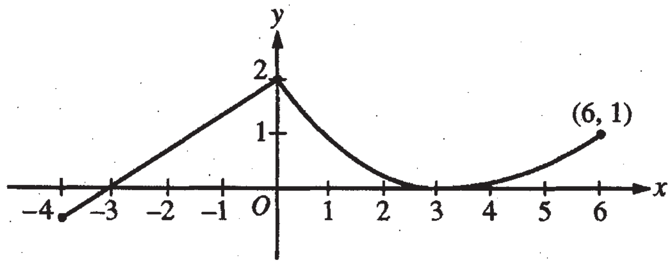

Graph of f

Work for problem 3(a)

No. $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $\Rightarrow$ nonexistent.

Work for problem 3(b)

$$\frac{\int_a^6 f'(x)\,dx}{6-a} = \frac{f(6)-f(a)}{6-a} = 0.$$

$f(6)=f(a)=1$, $a\neq 6$

$\therefore$ 2.

Continue problem 3 on page 9.

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Work for problem 3(c)

$$\frac{f(6)-f(a)}{6-a} = f'(c) = \frac{1}{3}$$

Yes. f is differentiable at all points of $0<x<6$. $-4<x<0$.

$\therefore$ There exists a "c" ~~test~~ at which point $f'(c)=\frac{1}{3}$

Work for problem 3(d)

$g''(x)>0$

$g'(x) = f(x)$

$g''(x) = f'(x)$

$f'(x)>0$.

$\therefore -3\leq x<0, \quad 3<x\leq 6$.

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END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

Question 3

Sample: 3A
Score: 9

The student earned all 9 points. Note that in part (c) the student affirms the hypotheses of the Mean Value Theorem, but generally that was not required to earn the second point. In part (d) the student earned the first point implicitly via $g''(x) = f'(x)$.

Sample: 3B
Score: 6

The student earned 6 points: no points in part (a), 2 points in part (b), 2 points in part (c), and 2 points in part (d). In part (a) the student is not working with a difference quotient. The answer is correct, but the justification is insufficient. In part (b) the student's work is correct. In part (c) the student earned both points even though the statement that "there exists a c with $3 < c < 6$ " is not included *and* the student may be implying that f is differentiable at $x = 0$. In part (d) the student earned the first 2 points. The student implicitly connects g' and f via $g''(x) = f'(x)$. The student makes the common error of using $f(0)$, instead of 0, as the right-hand endpoint of one of the intervals.

Sample: 3C
Score: 4

The student earned 4 points: no points in part (a), 2 points in part (b), no points in part (c), and 2 points in part (d). In part (a) the student is working with a difference quotient but not at $x = 0$. The answer is correct, but the justification is insufficient. In part (b) the student's work is correct. In part (c) the student never identifies $a = 3$. In part (d) the student earned the first 2 points, but the answer is not correct. Note that students were not penalized for including the endpoints in the correct intervals.

AP® CALCULUS AB
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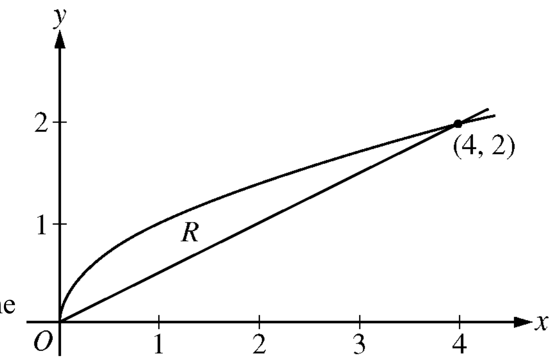
Question 4

Let R be the region bounded by the graphs of $y = \sqrt{x}$ and $y = \frac{x}{2}$, as shown in the figure above.



(a) Find the area of R.

(b) The region R is the base of a solid. For this solid, the cross sections perpendicular to the x-axis are squares. Find the volume of this solid.

(c) Write, but do not evaluate, an integral expression for the volume of the solid generated when R is rotated about the horizontal line $y = 2$.

(a) Area $= \int_0^4 \left(\sqrt{x} - \frac{x}{2}\right) dx = \frac{2}{3}x^{3/2} - \frac{x^2}{4}\Big|_{x=0}^{x=4} = \frac{4}{3}$

$3 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

(b) Volume $= \int_0^4 \left(\sqrt{x} - \frac{x}{2}\right)^2 dx = \int_0^4 \left(x - x^{3/2} + \frac{x^2}{4}\right) dx$

$= \frac{x^2}{2} - \frac{2x^{5/2}}{5} + \frac{x^3}{12}\Big|_{x=0}^{x=4} = \frac{8}{15}$

$3 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

(c) Volume $= \pi \int_0^4 \left(\left(2 - \frac{x}{2}\right)^2 - \left(2 - \sqrt{x}\right)^2\right) dx$

$3 : \begin{cases} 1 : \text{limits and constant} \\ 2 : \text{integrand} \end{cases}$

NO CALCULATOR ALLOWED

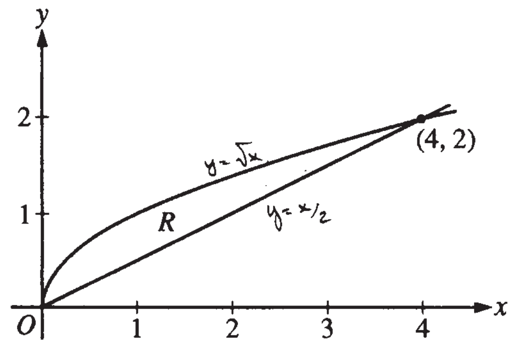
CALCULUS AB

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.



Work for problem 4(a)

$$R = \int_0^4 \left(\sqrt{x} - x/2\right) dx \Rightarrow \int_0^4 x^{1/2} - x/2 \, dx \Rightarrow \left[\frac{2x^{3/2}}{3} - x^2/4\right]_0^4$$

$$\Rightarrow \frac{2(4)^{3/2}}{3} - \frac{(4)^2}{4} \Rightarrow \frac{2\sqrt{64}}{3} - \frac{16}{4} \Rightarrow \frac{16}{3} - 4 \Rightarrow \frac{16-12}{3}$$

$$\Rightarrow \boxed{4/3}$$

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Continue problem 4 on page 11.

NO CALCULATOR ALLOWED

Work for problem 4(b)

$$V = \int_0^4 \left(\sqrt{x} - x/2\right)^2 dx \Rightarrow \int_0^4 x - x^{3/2} + x^2/4 \, dx \Rightarrow \left[x^2/2 - 2x^{5/2}/5 + x^3/12 \right]_0^4$$

$$\Rightarrow \left[\frac{4^2}{2} - \frac{2(4)^{5/2}}{5} + \frac{(4)^3}{12} \right] \Rightarrow \frac{16}{2} - \frac{2\sqrt{(4)^5}}{5} + \frac{64}{12} \Rightarrow 8 - \frac{64}{5} + \frac{16}{3} \Rightarrow \boxed{\frac{8}{15}}$$

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Work for problem 4(c)

$y = \sqrt{x} \Rightarrow x = y^2$ $\qquad$ $y = \frac{x}{2} \Rightarrow x = 2y$

$$V = \pi \int_0^4 \left(2 - x/2\right)^2 - \left(2 - \sqrt{x}\right)^2 dx$$

GO ON TO THE NEXT PAGE.

NO CALCULATOR ALLOWED

12×4 $\quad 12\overline{)48}$ $\quad 12 \times 4 = 48$

$16 \times 3 = 48$ $\quad 48^{1/2}$

$\frac{\cancel{16}\,\cancel{8}\,4}{4\;\cancel{2}\;1}$

16×16

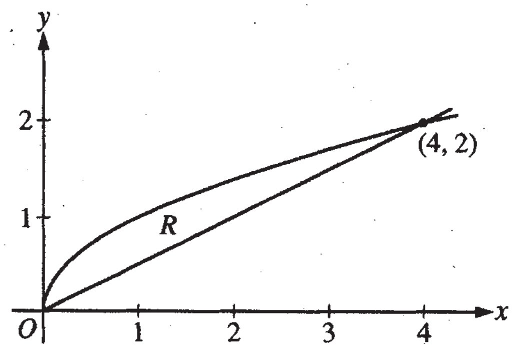
CALCULUS AB

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.



Work for problem 4(a)

a) $A_R = \int_0^4 \left(\sqrt{x} - \frac{x}{2}\right)dx = \int_0^4 \sqrt{x}\,dx - \int_0^4 \frac{x}{2}\,dx$

$\frac{2}{3}\sqrt[2]{x^3}\Big]_0^4 - \left[\frac{x^2}{\cancel{2}4}\right]_0^4 = \left[\frac{2}{3}\sqrt[2]{4^3} - 0\right] - \left[\frac{16}{4} - 0\right]$

$= \frac{2}{3}\sqrt[2]{48} - 4 =$

$1 \times \frac{24}{2}$

$12 \times 4 = 48$

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Continue problem 4 on page 11.

NO CALCULATOR ALLOWED

Work for problem 4(b)

$A_{square} = s^2 \qquad s = \sqrt{x} - \frac{x}{2}$

$$V = \int_0^4 s^2\,ds = \int_0^4 \left(\sqrt{x} - \frac{x}{2}\right)^2 dx = \int_0^4 \left(x - \frac{x^2}{4}\right) dx$$

$$= \left[\frac{x^2}{2} - \frac{x^3}{12}\right]_0^4 = \left[\left(8 - \frac{48}{12}\right) - 0\right] = 8 - 4 = 4\,u^3$$

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Work for problem 4(c)

axis of Rev. $\Rightarrow$ $y=2$ // to x-axis dx.

washer Method:

$R(x) = 2 - \frac{x}{2}$

$r(x) = 2 - \sqrt{x}$

$$V = \pi \int_0^4 \left[\left(2 - \frac{x}{2}\right)^2 - \left(2 - \sqrt{x}\right)^2\right] dx$$

GO ON TO THE NEXT PAGE.

 4C1

NO CALCULATOR ALLOWED

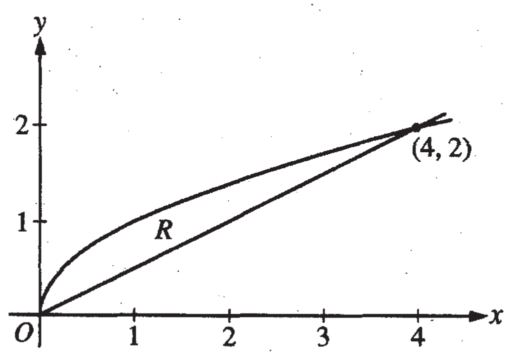
CALCULUS AB

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.



Work for problem 4(a)

$\sqrt{x} = 2$

$y_1 = \sqrt{x}$, $y_2 = \frac{x}{2}$, $y_1 = y_2 \Rightarrow \sqrt{x} = \frac{x}{2} \Rightarrow x = 4$

$\Rightarrow$ ~~at x = 2~~ Area $R = \int_0^4 \sqrt{x} - \frac{x}{2}\,dx$ $x^{1/2}$

$\Rightarrow$ Area $R = \frac{2}{3}x^{3/2} - \frac{x^2}{4}\Big]_0^4\,dx$

$\frac{2\times4\times2}{3} - \frac{16}{4} - 0 = \frac{16}{3} - \frac{16}{4} = \frac{16}{3} - 4 = \frac{16}{3} - \frac{12}{3}$

$= 4/3$ units^2

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Continue problem 4 on page 11.

NO CALCULATOR ALLOWED

Work for problem 4(b)

$V = s^3 \Rightarrow \cancel{\int_0^4 (\sqrt{x} - \frac{x}{2})^3}$ $\frac{d}{dx} \sqrt{x} - \frac{x}{2} = (\frac{1}{2\sqrt{x}} - \frac{1}{2})^3$

since square

$\Rightarrow \int_0^4 (\frac{1}{2\sqrt{x}} - \frac{1}{2})^3.$

Work for problem 4(c)

$V = \pi \int_0^4 R^2 - r^2 \, dx = \pi \int_0^4 (\sqrt{x})^2 - (\frac{x}{2})^2 dx. = \pi \int_0^4 x - \frac{x^2}{4} dx.$

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GO ON TO THE NEXT PAGE.

Question 4

Sample: 4A
Score: 9

The student earned all 9 points.

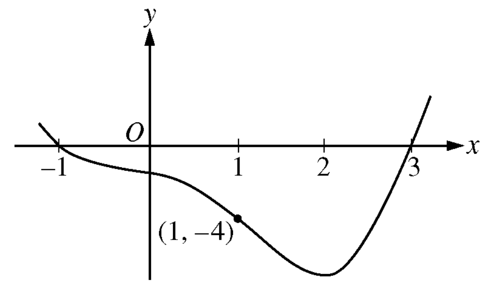
Sample: 4B
Score: 6

The student earned 6 points: 2 points in part (a), 1 point in part (b), and 3 points in part (c). In part (a) the student earned the first 2 points, but the answer is incorrect. In part (b) the student earned the integrand point. The student has an algebra error that leads to an incorrect antiderivative and was not eligible for the answer point. In part (c) the student's work is correct.

Sample: 4C
Score: 4

The student earned 4 points: 3 points in part (a), no points in part (b), and 1 point in part (c). In part (a) the student's work is correct. In part (b) the student has an incorrect integrand. In part (c) the student earned the limits and constant point.

Question 5



Graph of f'

Let f be a twice-differentiable function defined on the interval $-1.2 < x < 3.2$ with $f(1) = 2$. The graph of f', the derivative of f, is shown above. The graph of f' crosses the x-axis at $x = -1$ and $x = 3$ and has a horizontal tangent at $x = 2$. Let g be the function given by $g(x) = e^{f(x)}$.

(a) Write an equation for the line tangent to the graph of g at $x = 1$.

(b) For $-1.2 < x < 3.2$, find all values of x at which g has a local maximum. Justify your answer.

(c) The second derivative of g is $g''(x) = e^{f(x)}\left[(f'(x))^2 + f''(x)\right]$. Is $g''(-1)$ positive, negative, or zero? Justify your answer.

(d) Find the average rate of change of g', the derivative of g, over the interval $[1, 3]$.

(a) $g(1) = e^{f(1)} = e^2$ $g'(x) = e^{f(x)}f'(x)$, $g'(1) = e^{f(1)}f'(1) = -4e^2$ The tangent line is given by $y = e^2 - 4e^2(x-1)$.	$3 : \begin{cases} 1 : g'(x) \\ 1 : g(1) \text{ and } g'(1) \\ 1 : \text{tangent line equation} \end{cases}$
(b) $g'(x) = e^{f(x)}f'(x)$ $e^{f(x)} > 0$ for all x So, g' changes from positive to negative only when f' changes from positive to negative. This occurs at $x = -1$ only. Thus, g has a local maximum at $x = -1$.	$2 : \begin{cases} 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$
(c) $g''(-1) = e^{f(-1)}\left[(f'(-1))^2 + f''(-1)\right]$ $e^{f(-1)} > 0$ and $f'(-1) = 0$ Since f' is decreasing on a neighborhood of -1, $f''(-1) < 0$. Therefore, $g''(-1) < 0$.	$2 : \begin{cases} 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$
(d) $\dfrac{g'(3) - g'(1)}{3 - 1} = \dfrac{e^{f(3)}f'(3) - e^{f(1)}f'(1)}{2} = 2e^2$	$2 : \begin{cases} 1 : \text{difference quotient} \\ 1 : \text{answer} \end{cases}$

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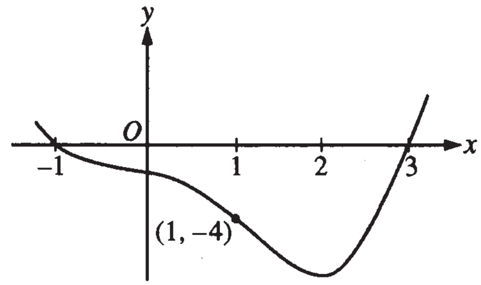

Graph of f'

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Work for problem 5(a)

$g'(1) = e^{f(1)} \cdot f'(1) = -4e^2$

$e^2 = -4e^2 + c$

$\Rightarrow c = 5e^2$

Equation of tangent line to g at $x = 1$:

$y = -4e^2 x + 5e^2$ //

Work for problem 5(b)

g has a local maximum when g' changes sign from positive to negative.

$g'(x) = e^{f(x)} \cdot f'(x)$

$e^{f(x)}$ is always positive, $\therefore$ $g'(x)$ changes sign from positive to negative when $f'(x)$ does so.

$f'(x)$ changes sign from positive to negative at $x = -1$.

$\therefore$ g has a local maximum at $x = -1$. //

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Continue problem 5 on page 13.

NO CALCULATOR ALLOWED

Work for problem 5(c)

$g''(x) = e^{f(x)}\left[(f'(x))^2 + f''(x)\right]$

$e^{f(x)}$ is always positive

$(f'(-1))^2 = 0$

$f''(-1)$ is negative.

$\therefore g''(-1)$ is negative.

Work for problem 5(d)

$g'(x) = e^{f(x)} \cdot f'(x)$

$g'(1) = e^{f(1)} \cdot f'(1) = -4e^2$

$g'(3) = e^{f(3)} \cdot f'(3) = 0$

Average rate of change $= \dfrac{0-(-4e^2)}{3-1}$

$= 2e^2$

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5B1

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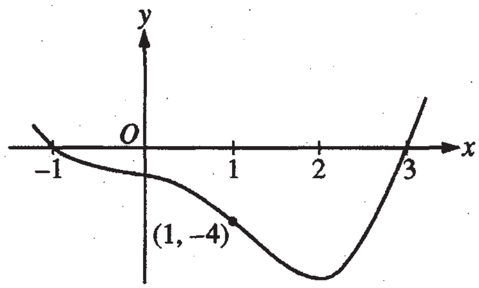

Graph of f'

Work for problem 5(a)

$g'(x) = e^{f(x)} \cdot f'(x)$

$g'(1) = e^{f(1)} \cdot f'(1) = e^2 \cdot -4 = -4e^2$

$g(1) = e^{f(1)} = e^2$

$\boxed{y - e^2 = -4e^2(x-1)}$

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Work for problem 5(b)

$g'(x) = e^{f(x)} \cdot f'(x)$

$e^{f(x)} \Rightarrow$ always positive

g has a local maximum at $x = -1$ because $g'(x)$ changes from positive to negative.

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Continue problem 5 on page 13.

Work for problem 5(c)

$g''(x) = e^{f(x)}\left[(f'(x))^2 + f''(x)\right]$

$g''(-1)$ is negative, $e^{f(x)}$ is positive because any e raised to any number is positive, $f'(-1) = 0$ (given) and $f''(-1) < 0$ (from the graph), so $g''(-1)$ is a positive * (zero + negative) which comes out to be a negative value,

Work for problem 5(d)

average rate of change of $g' = \frac{1}{3-1}\int_1^3 g''(x)\,dx$

$= \frac{1}{3-1} \cdot g'(x)\Big|_1^3 = \frac{g'(3) - g'(1)}{2}$

$g'(3) = e^{f(3)} \cdot f'(3) = e^{f(3)} \cdot 0 = 0$

$g'(1) = -4e^2$ (from 5(a))

$$= \frac{0 - (-4e^2)}{2} = \frac{4e^2}{2} = \boxed{2e^2}$$

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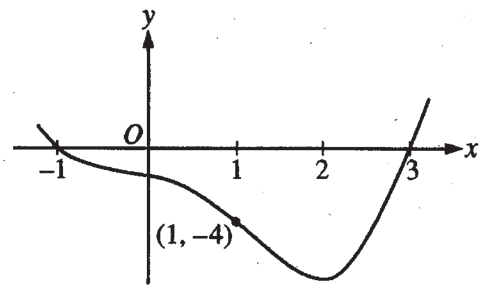

Graph of f'

Work for problem 5(a)

$g'(x) = e^{f(x)} \cdot f'(x)$

$g'(1) = e^{2} \cdot (-4)$

$= -4e^{2}$

$(y - e) = -4e^{2}(x - 1)$

Work for problem 5(b)

$f(x)$ has local max at $x = -1$

$g(x) = e^{f(x)}$

$\therefore g(x)$ has local max at $x = -1$

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Continue problem 5 on page 13.

Work for problem 5(c)

$f'(-1) = 0$

$f''(-1) < 0$ since $f'(x)$ is decreasing from $[-1/2, 2]$

since $f(1) = 2$ and $f(x)$ has only decreased from $f(-1)$ to $f(1)$, $f(-1) > 0$

$g''(x) = e^{f(x)}(0 + f''(x))$

$\therefore g''(-1) < 0$

Work for problem 5(d)

$g'(x) = e^{f(x)} \cdot f'(x)$

$g(1) = e^2 \cdot -4$

$= -4e^2$

$g(3) = 0$

avg rate of change $= \dfrac{g(3) - g(0)}{3-0}$

$= \dfrac{0 + 4e^2}{3}$

$= \dfrac{4e^2}{3}$

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Question 5

Sample: 5A
Score: 9

The student earned all 9 points. Note that in part (a) the student's first line earned the point for $g'(x)$. The student includes $g(1)$ implicitly in the second equation. In part (c) the justification is sufficient although the student does not explain why $f''(-1)$ is negative.

Sample: 5B
Score: 6

The student earned 6 points: 3 points in part (a), 1 point in part (b), 2 points in part (c), and no points in part (d). In part (a) the student's work is correct. In part (b) the student earned the answer point, but the justification is insufficient. The student does not describe the sign change in g'. In part (c) the student's work is correct. In part (d) the student is not working with the correct difference quotient.

Sample: 5C
Score: 3

The student earned 3 points: 1 point in part (a), 1 point in part (b), 1 point in part (c), and no points in part (d). In part (a) the student earned the point for $g'(x)$. The student does not have a value for $g(1)$. As a result, the second point was not earned, and the student was not eligible for the third point. In parts (b) and (c) the student earned the answer points. Both justifications are insufficient. In part (d) the student is not working with the correct difference quotient.

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Question 6

t (seconds)	0	8	20	25	32	40
$v(t)$ (meters per second)	3	5	−10	−8	−4	7

The velocity of a particle moving along the x-axis is modeled by a differentiable function v, where the position x is measured in meters, and time t is measured in seconds. Selected values of $v(t)$ are given in the table above. The particle is at position $x = 7$ meters when $t = 0$ seconds.

(a) Estimate the acceleration of the particle at $t = 36$ seconds. Show the computations that lead to your answer. Indicate units of measure.

(b) Using correct units, explain the meaning of $\int_{20}^{40} v(t)\,dt$ in the context of this problem. Use a trapezoidal sum with the three subintervals indicated by the data in the table to approximate $\int_{20}^{40} v(t)\,dt$.

(c) For $0 \le t \le 40$, must the particle change direction in any of the subintervals indicated by the data in the table? If so, identify the subintervals and explain your reasoning. If not, explain why not.

(d) Suppose that the acceleration of the particle is positive for $0 < t < 8$ seconds. Explain why the position of the particle at $t = 8$ seconds must be greater than $x = 30$ meters.

---

1 : units in (a) and (b)

(a) $a(36) = v'(36) \approx \dfrac{v(40) - v(32)}{40 - 32} = \dfrac{11}{8}$ meters/sec^2

1 : answer

(b) $\int_{20}^{40} v(t)\,dt$ is the particle's change in position in meters from time $t = 20$ seconds to time $t = 40$ seconds.

$$\int_{20}^{40} v(t)\,dt \approx \frac{v(20)+v(25)}{2}\cdot 5 + \frac{v(25)+v(32)}{2}\cdot 7 + \frac{v(32)+v(40)}{2}\cdot 8$$
$$= -75 \text{ meters}$$

$3 : \begin{cases} 1 : \text{meaning of } \int_{20}^{40} v(t)\,dt \\ 2 : \text{trapezoidal approximation} \end{cases}$

(c) $v(8) > 0$ and $v(20) < 0$
$v(32) < 0$ and $v(40) > 0$
Therefore, the particle changes direction in the intervals $8 < t < 20$ and $32 < t < 40$.

$2 : \begin{cases} 1 : \text{answer} \\ 1 : \text{explanation} \end{cases}$

(d) Since $v'(t) = a(t) > 0$ for $0 < t < 8$, $v(t) \ge 3$ on this interval.

Therefore, $x(8) = x(0) + \int_0^8 v(t)\,dt \ge 7 + 8 \cdot 3 > 30$.

$2 : \begin{cases} 1 : v'(t) = a(t) \\ 1 : \text{explanation of } x(8) > 30 \end{cases}$

NO CALCULATOR ALLOWED

t (seconds)	0	8	20	25	32	40
$v(t)$ (meters per second)	3	5	−10	−8	−4	7

Work for problem 6(a)

$$a(36) \simeq \frac{v(40)-v(32)}{40-32}$$

$$= \frac{7-(-4)}{8} = \frac{11}{8} \ m/s^2$$

Work for problem 6(b)

$$\int_{20}^{40} v(t)\,dt \simeq 5\left(\frac{-8+(-10)}{2}\right) + 7\left(\frac{-4+(-8)}{2}\right) + 8\left(\frac{7+(-4)}{2}\right)$$

$$= 5(-9) + 7(-6) + 8\left(\tfrac{3}{2}\right)$$

$$= -45 - 42 + 12 = -75 \text{ meters}$$

$\boxed{\text{total, not net}}$

$\int_{20}^{40} v(t)\,dt$ is the total displacement of the particle from $t = 20$ seconds to $t = 40$ seconds

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Continue problem 6 on page 15.

Work for problem 6(c)

Since $v(t)$ is differentiable, $v(t)$ is continuous.

Particle changes direction $\rightarrow$ $v(t)$ changes sign.

The particle must change direction in $(8,20)$ and in $(32,40)$.

$\left.\begin{array}{l} v(8)=5>0 \\ v(20)=-10<0 \end{array}\right\}$ $v(t)$ changes sign at some c for $8<c<20$

$v(t)=0$

$\left.\begin{array}{l} v(32)=-4<0 \\ v(40)=7>0 \end{array}\right\}$ $v(t)$ changes sign at some d for $32<d<40$

The above is true due do Intermediate Value Theorem.

Since $v(t)$ changes sign in $(8,20)$ and in $(32,40)$, the particle must change direction in $(8,20)$ and in $(32,40)$

Work for problem 6(d)

$a(t)>0$ for $0<t<8$ seconds.

Thus, $v(t)$ is increasing for $0<t<8$ seconds.

Since $v(0)=3$ m/s, and $v'(t)>0$ on $0<t<8$, absolute minimum of $v(t)$ on $0<t<8$ is 3 m/s

At 3 m/s, minimum distance travelled from $t=0$ to $t=8$

is $\int_0^8 v(t)\,dt = \int_0^8 3\,dt = 3\times 8 = 24$ metres.

$x(8)=x(0)+\int_0^8 v(t)\,dt = 7+\int_0^8 v(t)\,dt$.

Since, $\int_0^8 v(t)\,dt \geq 24$ metres, $x(8) \geq 31$ metres and $31>30$.

Thus, position of particle at $t=8$ seconds must be greater than $x=30$ metres

GO ON TO THE NEXT PAGE.

NO CALCULATOR ALLOWED

t (seconds)	0	8	20	25	32	40
$v(t)$ (meters per second)	3	5	−10	−8	−4	7

Work for problem 6(a)

$$a(36) = \frac{V(40) - V(32)}{40 - 32} = \frac{11}{8} \text{ meters per second square}$$

Work for problem 6(b)

$$\frac{L(20) + L(25)}{2}(25-20) + \frac{L(25) + L(32)}{2}(32-25) + \frac{L(40) + L(32)}{2}(40-32)$$

$$= \frac{-10-8}{2}\cdot 5 + \frac{-8-4}{2}\cdot 7 + \frac{-4+7}{2}\cdot 8 = -75 \text{ m}$$

This is the distance that the particle traveled during $20 < t < 40$, which is 75m left.

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Continue problem 6 on page 15.

NO CALCULATOR ALLOWED

Do not write beyond this border.

Work for problem 6(c)

Yes. In $8 < t < 10$ and $32 < t < 40$

Because the velocity changes from positive to negative during those subintervals.

Work for problem 6(d)

Because the acceleration of the particle is postive for $0 < t < 8$, so the velocity of the particle must be increasing from $t = 0$ to $t = 8$, from 3 m/s to 5 m/s.

Suppose the velocity is 3 m/s, after 8 seconds. The particle will travel 24 meters.

24 meters plus the initial 7 meters is 31 meters.

So, by using the ~~smallest~~ ~~amount~~ slowest of velocity, the car still can travel more than 30 meters.

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 6C1

NO CALCULATOR ALLOWED

t (seconds)	0	8	20	25	32	40
$v(t)$ (meters per second)	3	5	–10	–8	–4	7

Work for problem 6(a)

By the Mean Value Theorem:

$$a(36) = \frac{v(40) - v(32)}{40 - 32} = \frac{11}{8} \text{ (meters/seconds}^2\text{)}$$

Work for problem 6(b)

$\int_{20}^{40} v(t) \cdot dt$ shows us the overall sum of changes of $v(t)$ during 20 seconds from $t=20$ to $t=40$.

$$\int_{20}^{40} v(t)dt \approx (9 \cdot 5 + 6 \cdot 7 + 8 \cdot 9) = 45 + 42 + 72 = 159$$

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Continue problem 6 on page 15.

NO CALCULATOR ALLOWED

Work for problem 6(c)

Yes, It must change the direction on the given intervals $t \in (8;20)$ and $t \in (32;40)$, Because velocity changes its sign on these intervals

Work for problem 6(d)

As $V'(t) = a(t)$, $a(t)$ is positive for $0 < t < 8$ seconds, $V(t)$ is also positive, therefore, $V(t)$ is increasing for $0 < t < 8$ seconds,

$X'(t) = V(t)$

$X(8) = X(0) + \int_0^8 (V(t) \cdot dt)$; $X(0) = 7$, $\int_0^8 v(t) \cdot dt$ is more than 23 (32, for instance, using trapezoidal rule),

so, $X(8) > 30$ meters

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Question 6

Sample: 6A
Score: 9

The student earned all 9 points. Note that in part (b) students could include units in either the numerical answer or the verbal description. The student's use of "total" is not necessary.

Sample: 6B
Score: 6

The student earned 6 points: the units point, 1 point in part (a), 2 points in part (b), no points in part (c), and 2 points in part (d). In part (a) the student's answer is correct. The use of an equality sign instead of an approximation symbol was ignored. In part (b) the student did not earn the point for the meaning of the definite integral, because the response uses "distance" instead of net distance. The student earned 2 points for the trapezoidal approximation; the use of L instead of v was ignored. In part (c) the student has only one correct interval, and the justification is inconsistent with that correct interval. The student was eligible for a point only if the justification matched the correct interval. In part (d) the student's work is correct. The verbal argument notes that the velocity is increasing, implies that $v(t) \geq 3$ on the interval, and argues from the initial position plus distance traveled.

Sample: 6C
Score: 4

The student earned 4 points: no units point, 1 point in part (a), no points in part (b), 2 points in part (c), and 1 point in part (d). In part (a) the student's work is correct. In part (b) the student does not include units and is not using a trapezoidal approximation. In part (c) the student's work is correct. The student was not required to describe the nature of the sign changes in $v(t)$. In part (d) the student earned the first point. There is no valid explanation as to why the definite integral is more than 23. The student needs to appeal to the fact that $v(t) \geq 3$ for $0 < t < 8$.